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Microwave background fluctuations and dark matter

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Limits on the anisotropy of the microwave background provide strong constraints on theories of galaxy formation which incorporate non-baryonic dark matter. We focus on scale-invariant perturbations, which may be in either an adiabatic or an isocurvature mode. Adiabatic models with cold dark matter in which galaxies trace the mass distribution lead to excessive small-scale anisotropies unless $\Omega_0 h_0^{4/3} > 0.2$. This apparently conflicts with the low value of Ω_0 deduced from dynamical studies of galaxy clustering. This difficulty may be resolved if galaxies are biased tracers of the mass. Isocurvature cold dark-matter models are incompatible with observations even if $\Omega_0 = 1$ unless the amplitude of the galaxy correlation function is more than four times that of the mass distribution. The statistics of the radiation pattern may provide a useful test of the Gaussian nature of the fluctuations.

1. Introduction

It is now over twenty years since the microwave background radiation was discovered. From then until now, experiments designed to detect anisotropies have steadily achieved higher and higher levels of sensitivity, yet no relic fluctuations have been seen. This is potentially embarrassing, for if the large-scale structure in our Universe were formed by gravitational instability, then the cosmic microwave background radiation ought to display residual fluctuations.

However, the expected amplitude of the microwave background anisotropies is sensitive to the nature of the dark matter in the Universe, as well as the cosmological parameters Ω_0 and H_0. Present upper limits are small enough to severely constrain the gravitational instability theory if the density of the Universe were dominated by baryonic matter (Wilson & Silk 1981; Kaiser 1983). Bond *et al.* (1980) and Doroshkevich *et al.* (1980) suggested that this problem could be resolved if the Universe were dominated by massive neutrinos. This idea has now lost popularity because *N*-body simulations of the neutrino model have exposed serious problems arising from the large coherence length ($\lambda \approx 17(\Omega_0 h_0^2)^{-1}$ Mpc)† of the post-recombination fluctuation spectrum (White *et al.* 1983, 1984; Frenk 1986, this symposium).

Cold dark matter (e.g. axions, photinos, gravitinos, etc.) offers an attractive alternative to massive neutrinos (see Blumenthal *et al.* 1984, for a general review). If cold dark matter (CDM) models are to prove acceptable, they should at least explain the clustering of galaxies without violating any limits on anisotropies in the microwave background. *N*-body simulations have been used to study clustering in CDM models (Davis *et al.* 1985). However, as described by Frenk (1986, this symposium) the results are ambiguous because there is no guarantee that the luminous material accurately traces the distribution of the dark matter. If galaxies follow the mass distribution, then CDM simulations with $\Omega_0 \approx 0.2$ match many features of the galaxy

† h_0 is Hubble's constant (H_0) in units of 100 km s^{-1} Mpc^{-1} (1 pc ≈ 30857 × 10^{12} m).

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distribution and are compatible with dynamical estimates of the mean mass density, which imply that $\Omega_0 = 0.14 \times 2^{\pm 1}$ (Davis & Peebles 1983; Bean *et al.* 1983). CDM simulations with $\Omega_0 = 1$ can also yield acceptable fits, but only if galaxies formed preferentially in high-density regions.

As we will show in this review, experimental limits on small-angle anisotropies provide powerful constraints on low-density CDM models. Further, anisotropy limits on large angular scales set constraints on CDM models with isocurvature initial conditions. The microwave background observations therefore lead us to favour high density ($\Omega_0 \approx 1$) cold dark matter models with adiabatic initial conditions.

Our results are based on a number of assumptions, which are summarized in §2. Adiabatic and isocurvature perturbations are discussed in §§3 and 4 respectively. In §5 we briefly describe some aspects of the statistical properties of the radiation field. Our main conclusions are presented in §6.

2. Assumptions

1. We assume a metric

$$ds^2 = a^2(\tau)(\eta_{ij} + h_{ij})\, dx^i\, dx^j, |h_{ij}| \ll 1, \tag{1}$$

where $a(\tau)$ is the scale factor, $\tau = \int dt/a$ is conformal time, and $\eta_{ij} = (1, -1, -1, -1)$ is the background metric. The flat background metric restricts our analysis of low density models to angular scales on which curvature of the spacelike hypersurfaces can be neglected $\theta < \frac{1}{2}\Omega_0(1-\Omega_0)^{-\frac{1}{2}}$ (see also point 3). The cosmological constant Λ is assumed to be zero.

2. The primordial perturbations may be in either an adiabatic or an isocurvature mode. Precise definitions of the two modes will be given in §§3 and 4 respectively. The density fluctuations will be assumed to be Gaussian, as predicted in a wide class of cosmological models. In particular, this is the case if the fluctuations are quantum mechanical in origin, generated as zero point oscillations of scalar fields in de Sitter space.

3. The primordial fluctuation spectrum will be assumed to have the Harrison–Zel'dovich scale-invariant form (Harrison 1970; Zel'dovich 1972). This type of spectrum is predicted in inflationary models (for both adiabatic and isocurvature fluctuations), but so far acceptable amplitudes have been obtained only in certain specific particle physics models (see, for example, Bardeen *et al.* 1983). However, the prediction of a scale invariant spectrum is more general and not restricted exclusively to inflationary models (see, for example, Kibble 1976; Press 1980). Note that in low-density models there is no natural generalization of the scale-invariant spectrum on scales exceeding that of the spatial curvature.

4. We assume that the primeval plasma recombined at the usual redshift $z_{\rm rec} \approx 1000$ (Peebles 1968) and that any reheating of the intergalactic medium occurred too late to cause significant rescattering of the microwave background photons. If re-ionization occurs at redshift z_*, then the optical depth from Thomson scattering is

$$\tau_{\rm opt} = 3.6 \times 10^{-2}\, x_{\rm e}\, \Omega_{\rm B}\, h_0/\Omega_0^2\, [2 - 3\Omega_0 + (1+\Omega_0 z_*)^{\frac{1}{2}}(\Omega_0 z_* + 3\Omega_0 - 2)], \tag{2}$$

where $x_{\rm e}$ is the degree of ionization (assumed to be constant) and $\Omega_{\rm B}$ is the cosmological density in baryons. For representative parameters (e.g. $\Omega_0 \approx 1$, $\Omega_{\rm B} \approx 0.1$, $h_0 \approx 0.75$), rescattering would have erased microwave background anisotropies on small angular scales ($\theta \lesssim (\Omega_0/z_*)^{\frac{1}{2}}$) only if reheating occurred at high redshift ($z_* \gtrsim 50$). In §3, we argue that this is unlikely if the initial fluctuations were scale-invariant.

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5. The amplitude of the initial fluctuations is fixed by matching the second moment of the mass autocovariance function,

$$J_3(x_0) = \int_0^{x_0} \xi(x)\, x^2\, \mathrm{d}x, \tag{3}$$

to observational estimates from the Center for Astrophysics (CfA) redshift survey (Davis & Peebles 1983). If x_0 is chosen so that $\xi(x_0) \ll 1$, then $J_3(x_0)$ should be insensitive to nonlinear clustering on small scales (Peebles 1981). Accordingly, we choose $x_0 = 10\, h_0^{-1}$ Mpc, for which we deduce $J_3(10\, h_0^{-1}) = 270\, h_0^{-3}\, \mathrm{Mpc}^3$ from the CfA survey. In normalizing our calculations in this way, we are assuming that galaxies are accurate tracers of the mass distribution.

Some of these assumptions may appear unduly restrictive. For example, proponents of inflation and an $\Omega_0 = 1$ universe might argue strongly against assumption (5) because a high-density universe is incompatible with the dynamics of galaxy clustering unless galaxies are more highly clustered than the mass distribution (see §1). Advocates of models featuring cosmic strings might argue against assumption (2) because strings produce a non-Gaussian density field (Vilenkin 1981). However, in the absence of any direct conflict with observations these assumptions form a well defined starting point. As various observations (including background anisotropy limits) lead to contradictions, we can reexamine our assumptions and revise them accordingly.

3. Adiabatic perturbations

The most interesting constraints on adiabatic CDM models, subject to the assumptions listed in §2, are set by the anisotropy experiment of Uson & Wilkinson (1984*a*, *b*). These authors report a 95% confidence limit of $\Delta T/T < 2.9 \times 10^{-5}$ at an angular scale of 4.5′. This angular separation corresponds to a comoving length of $\lambda_0 = 7.9(\Omega_0 h_0)^{-1}$ $(\theta/4.5')$ Mpc and is comparable to the angle subtended by the horizon at the time of decoupling ($\theta \sim 30\Omega_0^{\frac{1}{2}}{}'$). On these scales, acoustic fluctuations will be damped as the universe becomes optically thin and an accurate treatment of the fluctuations requires a full numerical solution of the Boltzmann transport equations for the photons. The relevant equations and details of the numerical techniques may be found in various papers and will not be repeated here (see Peebles & Yu 1970; Wilson & Silk 1981; Bond & Efstathiou 1984; Vittorio & Silk 1984). Large-angle anisotropies in this model have been considered by Peebles (1982).

The main physical effects are illustrated in figure 1, which shows the evolution of plane waves with comoving wavenumbers $k = (10\ \mathrm{Mpc})^{-1}$ (figure 1*a*) and $k = (1\ \mathrm{Mpc})^{-1}$ (figure 1*b*). (We use the usual synchronous gauge, $h_{0i} = 0$). These diagrams show the energy density fluctuations in four components: CDM (δ_x), baryons (δ_B), photons (δ_γ) and massless neutrinos (δ_ν). The initial conditions correspond to adiabatic perturbations,

$$\delta_\mathrm{x}(k, \tau_i) = \delta_\mathrm{B} = \tfrac{3}{4}\delta_\gamma = \tfrac{3}{4}\delta_\nu = A(k),\ k\tau_i \ll 1. \tag{4}$$

Because we assume scale-invariant perturbations, the initial power spectrum is $P(k) = |A(k)|^2 \propto k$. While the scale of the perturbations exceed the horizon ($k\tau \ll 1$), each component grows as $\delta \propto \tau^2$ (see, for example, Peebles 1980, chapter 5). When the perturbations enter the horizon ($k\tau > 1$) the photons and baryons, which are tightly coupled by Thomson scattering, begin to oscillate acoustically ($\delta \propto \exp{(\mathrm{i}k\tau/\sqrt{3})}$) and the massless neutrino perturbations decay rapidly because of phase mixing (Peebles 1973; Bond & Szalay 1983). The

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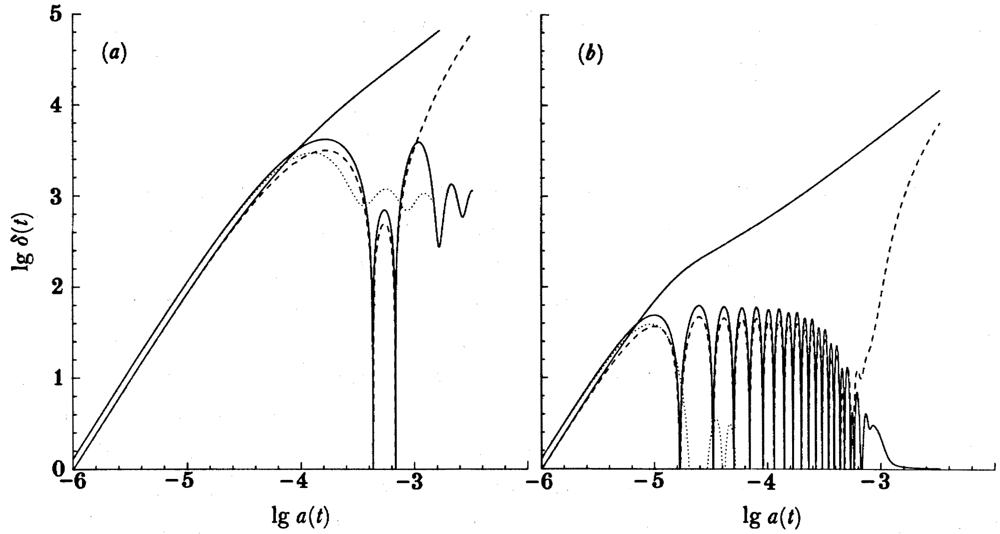

FIGURE 1. Evolution of adiabatic plane wave perturbations in a CDM universe with $\Omega_0 = 1$, $\Omega_B = 0.03$, $H_0 = 1$; *a* corresponds to the comoving wavenumber $k = (10\text{ Mpc})^{-1}$ and *b* shows $k = (1\text{ Mpc})^{-1}$. The monotonically increasing solid lines show the evolution of perturbations in the cold dark matter, dashed lines show the baryon perturbation, dotted lines show the evolution of massless neutrinos (which are switched off when they become dynamically unimportant) and oscillating solid lines show the photon perturbations.

CDM fluctuations continue to grow when the fluctuations enter the horizon, though the growth rate is reduced during the radiation era (Meszaros 1975). When the Universe becomes matter dominated ($z < z_{eq} = 2.5 \times 10^4 \omega_0 h_0^2$) the CDM fluctuations once again grow as $\delta_x \propto \tau^2$. At recombination ($z_{rec} \approx 1000$) the baryons decouple from the photons and rapidly fall into the potential wells created by the CDM fluctuations. Thus, at $z < z_{rec}$ the baryons should accurately follow the distribution of the cold component, though this need not apply once nonlinear structures form which can inject energy into the gas.

As figure 1*b* shows, short-wavelength perturbations in the photons are severely damped before recombination is complete. Well after recombination, the photon perturbations continue to oscillate, but the oscillations are offset from zero by a constant amount $2\,\ddot{\delta}_x/k^2$ (where dots denote differentiation with respect to τ). This represents the gravitational effect of the matter on the radiation ($\delta_\gamma \approx \Delta\phi/c^2 \approx (\delta\rho/\rho)_e\, l^2$). Although the radiation density is inhomogeneous at late times, this effect does not lead directly to observable small-scale anisotropies in the microwave background. The main goal of the numerical integrations of the Boltzmann equation is to compute the residual amplitude in higher angular moments of the photon distribution function.

Let us define the correlation function of the temperature fluctuations,

$$C(\theta, \tau_0) = \langle \Delta T/T(\hat{q}, x, \tau_0)(\Delta T/T(\hat{q}', x, \tau_0)\rangle, \quad \cos\theta = \hat{q}\cdot\hat{q}', \tag{5}$$

(where angular brackets denote an ensemble average). On small angular scales ($\theta < 100'$), the numerical results of Bond & Efstathiou (1984) can be well approximated by the simple formula,

$$C(\theta)/C(0) = [1 + (\theta/\theta_c)^2/(2\beta)]^{-\beta}, \tag{6a}$$

where

$$\beta \approx 1,$$
$$\theta_c \approx (8.5\Omega_0^{\frac{1}{2}})', \tag{6b}$$
$$C(0)^{\frac{1}{2}} \approx 7.5 \times 10^{-6}\,\Omega_0^{-1.2}\,h_0^{-1.6}.$$

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The parameters in (6*b*) are approximately valid (to within *ca.* 30%) for CDM universes in which $\Omega_B \sim 0.03$. $C(0)$ has been normalized to the observed amplitude of galaxy clustering as described in §2. For more accurate values of the parameters in (6), and for fits to the postrecombination fluctuation spectrum of the cold component, see table 1 of Bond & Efstathiou (1984).

Uson & Wilkinson (1984*a*, *b*) have observed 12 well-separated regions of the sky. In each patch, they compute the temperature difference $T_F - \frac{1}{2}(T_1 + T_2)$, where T_F is the temperature in the direction of a target field, F, and $\frac{1}{2}(T_1+T_2)$ is the mean temperature in two reference fields located 4.5′ on either side of the target. For this experimental arrangement we predict that the temperature anisotropies should be Gaussian distributed, with variance

$$(\Delta T/T)^2 = 2[C(0,\sigma) - C(\theta,\sigma)] - \tfrac{1}{2}[C(0,\sigma) - C(2\theta,\sigma)], \tag{7}$$

where $C(\theta,\sigma)$ indicates that $C(\theta)$ has been convolved with a Gaussian response function of width σ to mimic the finite beam width of the experiment ($\sigma \approx 1.1'$).

Predictions for $\Delta T/T$ for various CDM models are shown in figure 2, together with Uson & Wilkinson's 95% upper limit. The two CDM models with $\omega_0 = 0.2$ are excluded by the observations, whereas the $\Omega_0 = 1$ CDM model predicts temperature fluctuations which lie well below current levels of sensitivity. Generally, CDM models must satisfy the constraint

$$\Omega_0 h_0^{\frac{4}{3}} > 0.2, \tag{8}$$

to remain compatible with Uson & Wilkinson's upper limit (Bond & Efstathiou, 1984). If we also require that the age of the Universe exceed the inferred ages of globular clusters ($t_0 \approx 15 \times 10^9$ a, see for example Sandage 1983), then (8) implies a high-density universe with $\Omega_0 \gtrsim 0.5$. This is outside the range allowed by dynamical estimates of Ω_0.

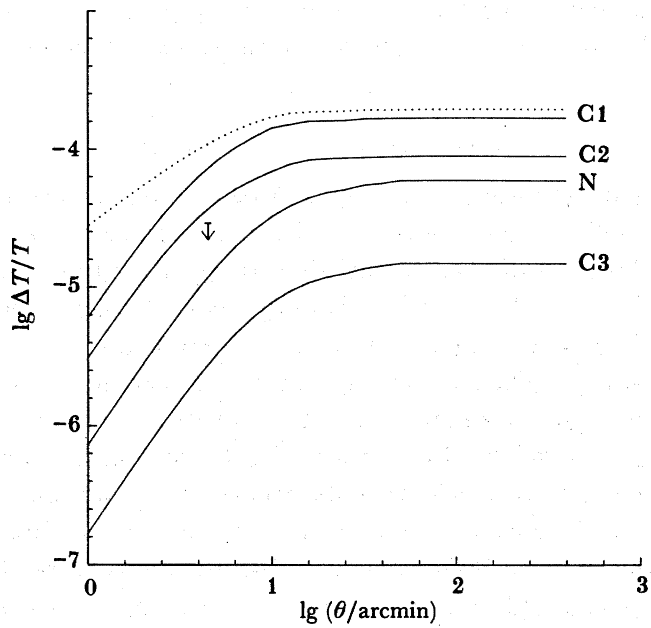

FIGURE 2. Temperature fluctuations as a function of angular scale. Curves C1–C3 show results for adiabatic CDM models with the following parameters: C1, $\Omega_0 = 0.2$, $H_0 = 0.5$; C2, $\Omega_0 = 0.2$, $H_0 = 0.75$; C3, $\Omega_0 = 1.0$, $H_0 = 0.75$. The solid lines show our predictions for the experimental arrangement used by Uson & Wilkinson (8). In case C1 we compare their procedure with the results expected in a beam switching experiment $\{(\Delta T/T)^2 = 2[C(0,\sigma) - C(\theta,\sigma)]\}$, which is shown as the dashed line. The curve labelled *N* shows an $\Omega_0 = 1, H_0 = 0.75$ model in which massive neutrinos dominate the present mass density. In all cases we have assumed $\Omega_B = 0.03$.

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The microwave background constraints therefore suggest that CDM models in which light traces mass are probably not correct because they result in excessive small-scale anisotropies (see also Vittorio & Silk 1984). These results point towards 'biased' galaxy formation models in which galaxies form only at high spots in the initial density field. Davis *et al.* (1985) have given a specific (though *ad hoc*) example of biasing in an $\Omega_0 = 1$ CDM universe which is compatible with the observed pattern and dynamics of galaxy clustering. The temperature fluctuations in such models present a considerable challenge to prospective experimenters. For example, Uson & Wilkinson's experiment would have to reach a precision of $\Delta T/T < 2.1 \times 10^{-6}$ to firmly exclude the model of Davis *et al.* (1985).

There are several points worth mentioning here. Firstly, these conclusions are based entirely on one experiment and on the quoted significance level. It is extremely important that more experiments be done. Second, the constraints require that re-ionization was not important in rescattering microwave background photons. With the scale-invariant spectrum assumed here only a small fraction of the baryonic material ($\Omega_B \approx 10^{-4}$–10^{-6}) could have collapsed into nonlinear systems by a redshift $z_*(\tau_{opt} = 1)$ (Bond & Efstathiou 1984, equation 2). Even if this material formed very massive stars with high efficiency it is only marginally possible that the Universe could have been fully re-ionized (Carr *et al.* 1984). Although significant rescattering seems unlikely, high-precision anisotropy experiments at intermediate angular scales ($\theta \gtrsim 1°$) would be highly desirable because these scales would not have been affected by re-ionization. Third, the predications of RMS temperature fluctuations do not depend on the assumption of random phases.

In figure 2 we also show predictions for a massive neutrino model with $\Omega_0 = 1$, $H_0 = 0.75$. As with the CDM models, we have assumed Gaussian scale-invariant adiabatic perturbations, but we have fixed the amplitude so that typical nonlinear structures form at a redshift $z_{nl} = 3$ (i.e. $\xi(0, z_{nl}) = 1$). As this figure shows, a modest improvement in the anisotropy limits would result in strong constraints on the neutrino model which would not be sensitive to the details of galaxy formation.

4. Isocurvature perturbations

It has become fashionable to assume that only adiabatic perturbations were present in the early Universe. The possibility of isocurvature ('isothermal') perturbations has lost popularity primarily because grand unified theories predict that irregularities in the ratio of photons to baryons would not have survived the epoch of baryosynthesis. However, both adiabatic and isocurvature scale-invariant perturbations can arise in inflationary models which are axion dominated at the present epoch. The relative amplitudes of the two modes are highly uncertain at present, but there is no guarantee that the adiabatic mode dominates as has usually been assumed (see, for example, Seckel & Turner 1985). In this section we assume that only the isocurvature mode is present and we calculate the large-angle anisotropies in the microwave background. A more detailed analysis of this mode is presented by Efstathiou & Bond (1986). Although our discussion is motived by the axion model, the results do not depend upon any specific details of the particle physics and apply to any cold dark-matter model with isocurvature perturbations.

To simplify the discussion, we assume a two-component universe containing axions with mean density ρ_a and 'photons' with mean density $\bar{\rho}_\gamma$ (the 'photon' component includes gluons, relativistic quarks, etc.). At temperatures greater than 1 GeV, the axion is massless and

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non-interacting and we assume that the universe is perfectly smooth. At $T \approx 1$ GeV (corresponding to time τ_i) the axion acquires a mass through a symmetry breaking associated with QCD confinement. Quantum fluctuations in the axion misalignment angle will lead to a spatially irregular mass distribution of the axion field at τ_i of amplitude

$$\delta_a(k, \tau_i) = A(k), \quad |A(k)|^2 \propto k^{-3}. \tag{9}$$

This spectrum corresponds to scale-invariant fluctuations (Axenides *et al.* 1983). Because energy must be conserved locally, there will be compensating fluctuations in the quark-gluon plasma of opposite sign to those in the axions

$$\delta_\gamma = -(\bar{\rho}_a/\bar{\rho}_\gamma)\,\delta_a \ll \delta_a, \quad \tau = \tau_i. \tag{10}$$

At τ_i the perturbed metric obeys the conditions

$$h_{ij}(\tau_i) = 0, \quad \dot{h}(\tau_i) = 0, \quad (h = -h_i^i). \tag{11}$$

Equations (10) and (11) specify the isocurvature mode. These initial conditions correspond to a perturbation in the equation of state of the universe; the curvature remains unchanged. The evolution of a long-wavelength perturbation ($k\tau \ll 1$) can easily be predicted by noting that the perturbation in the photon entropy per axion, $S_\gamma = \frac{3}{4}\delta_\gamma - \delta_a$, remains approximately constant while the perturbation lies outside the horizon. Because the perturbation is an isocurvature mode, the fluctuations in the axion component must decay as the universe becomes matter dominated ($\delta_a \rightarrow 0$ as $\tau \rightarrow \infty$), while the perturbation in the radiation must increase ($\delta_\gamma \rightarrow -\frac{4}{3}A(k)$ as $\tau \rightarrow \infty$). These radiation fluctuations can lead to substantial anisotropies in the microwave background radiation. For example, Efstathiou & Bond (1986) compute a quadrupole amplitude of

$$a_2 \approx 5 \times 10^{-5}\, h_0^{-1}, \quad \Omega_0 = 1 \tag{12}$$

(where the temperature pattern on the sky is written as $\Delta T/T = \sum a_l^m\, Y_l^m(\theta, \phi)$ and $\langle |a_2^m|^2 \rangle = a_2^2$). As for the adiabatic CDM models, the amplitude of the isocurvature perturbations has been fixed by using (3) and assuming that galaxies trace the mass. On large angular scales ($\theta \gtrsim 1°$) the radiation correlation function is predicted to be

$$C(\theta) \approx (3a_2^2/2\pi)\,\{\ln\,[2/(1-\cos\theta)] - 1 - \tfrac{3}{2}\cos\theta\} \tag{13}$$

after subtraction of the monopole and dipole components.

Equations (12) and (13) are to be compared with the 90% upper limits of $a_2 < 1.1 \times 10^{-4}$ (Fixsen *et al.* 1983; Lubin & Villela 1986) and $C(\theta)^{\frac{1}{2}} < 3.7 \times 10^{-5}$ over the range $10° < \theta < 180°$ (Fixsen *et al.* 1983). The constraints imposed by the upper limit on $C(10°)$ are particularly strong. Even if we invoke biased galaxy formation, an $\Omega_0 = 1$, $h_0 = 0.5$ model is only compatible with the observations if $J_3(10h_0^{-1}$ Mpc) for the mass distribution is nine times smaller than that of the galaxy distribution. Such a large ratio seems unreasonable (Bardeen *et al.* 1986). Models of $\Omega_0 = 1$ with $h_0 > 1$ and a moderate level of biasing may be compatible with the observations but would conflict with estimates of the age of the Universe. Similar arguments, together with limits imposed by small-scale anisotropies, can be used to severely constrain low-density isocurvature models (Efstathiou & Bond 1986).

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5. Statistics of the radiation pattern

The theoretical predictions described in the previous sections have been confined exclusively to RMS quantities. If the fluctuations are Gaussian, the statistical properties of the radiation pattern are completely specified by the temperature correlation function $C(\theta)$ or, equivalently, by the power spectrum $S(k)$. The Gaussian assumption allows us to make highly specific predictions concerning the texture of the temperature fluctuations. As an example, we have constructed a realization of the radiation pattern in an Ω_0^{-1} adiabatic CDM model (figure 3). Using the fit to $C(\theta)$ given in 6*a* with $\beta = 1$, we can derive a convenient expression for the power spectrum which is valid in the small-angle approximation (i.e. ignoring curvature of the celestial sphere)

$$S(k) \propto K_0(\sqrt{2}\, k\theta_c). \tag{14}$$

The map shown in figure 3 was generated by setting up a random phase realization of the power spectrum $S(k)$ and Fourier transforming to recover the sky brightness. The solid lines show contours at several thresholds above the RMS fluctuation level and dotted lines show contours at negative thresholds.

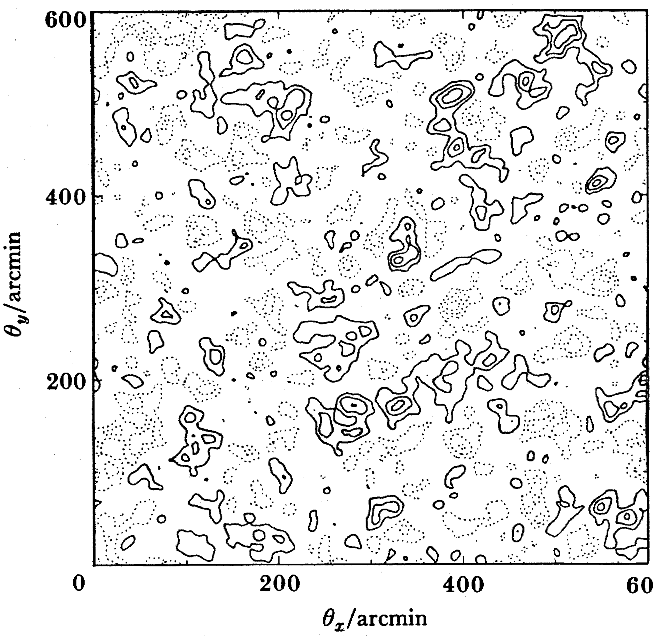

Figure 3. Contour map of the radiation pattern in an $\Omega_0 = 1$ CDM model with adiabatic Gaussian fluctuations. Solid lines show contours at thresholds $\nu = 1, 2, 3$ times the RMS amplitude. Contours at $\nu = -1, -2, -3$, are shown as the dotted lines.

This figure displays several interesting features. At low thresholds, the contours are highly elongated and irregular, but at high thresholds they become rounder and smoother. High peaks tend to occur in associations and are more strongly correlated than lower threshold peaks. These properties and a variety of other statistics can be computed analytically by applying techniques developed by Bardeen *et al.* (1986) for multidimensional Gaussian random fields (Bond & Efstathiou 1986). Some aspects of the statistics of very high peaks have been considered independently by Sazhin (1985).

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If such a map could be constructed for the real sky, it would provide an extremely powerful test of theories of galaxy formation. On small-angular scales we would learn about the coherence angle θ_c and thus deduce a constraint on Ω_0 (6*b*). The large-scale features would allow a direct test of the shape of the primordial spectrum and an important check of the Gaussian nature of the fluctuations.

6. Conclusions

The microwave background anisotropies represent one of our most powerful probes of the early Universe. We have shown that present upper limits place important constraints on galaxy formation schemes incorporating non-baryonic dark matter. The observations on small angular scales are at an interesting stage, since a modest improvement in sensitivity will strengthen our arguments concerning low-density cold dark-matter models and may firmly exclude neutrino-dominated models. We would also like to see improved experiments at angular scales greater than about 5°. These would set interesting limits on a scale-free perturbation spectrum without any ambiguities associated with re-ionization. The present upper limits on intermediate and large angular scales appear to exclude any reasonable isocurvature cold dark matter model. This demonstrates the power of microwave background experiments to eliminate relatively exotic possibilities from the theorists' list. The statistics of the radiation pattern may also prove fruitful. In particular, the temperature fluctuations should yield one of the strongest tests of whether the primordial perturbations were indeed Gaussian.

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